312 '03-MIDTERM 1

Name:____

September 30, 2003

1 $< 25pts >$

Solve the Laplace equation

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0 \ , \ \ 0 \le x \ \ y \le \pi \ ,$$

with BC

$$u(x,0) = u(x,\pi) = 0 \ , 0 \le x \le \pi \ ,$$

$$u(0,y) = u(\pi,y) = \sin y \ , 0 \le y \le \pi \ .$$

Solution

2 $< 25pts >$

Solve the heat equation

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}$$

on the interval $0 \leq x \leq \pi$ subject to the boundary conditionis $u(0,t) = \sin \omega t$, $u(\pi, t) = 0$ and initial condition

$$u(x,0) = \begin{cases} 1\,, & 0 < x < \pi/2 \\ 0\,, & \pi/2 < x < \pi \end{cases}$$

using a sine series expansion.
(Hint: multiply both sides of the equation by $\sin kx$ and integrate $\int_0^\pi$; use integration by parts carefuly!)
Solution

3 $< 25pts >$

If

$$\triangle u = 0 \text{ for } x^2 + y^2 < 1 \ ,$$
$$u(x,y) = xy \text{ for } x^2 + y^2 = 1$$

find $u(0,0)$ by using the mean value theorem.
(Hint: use polar coordinates!)
Solution

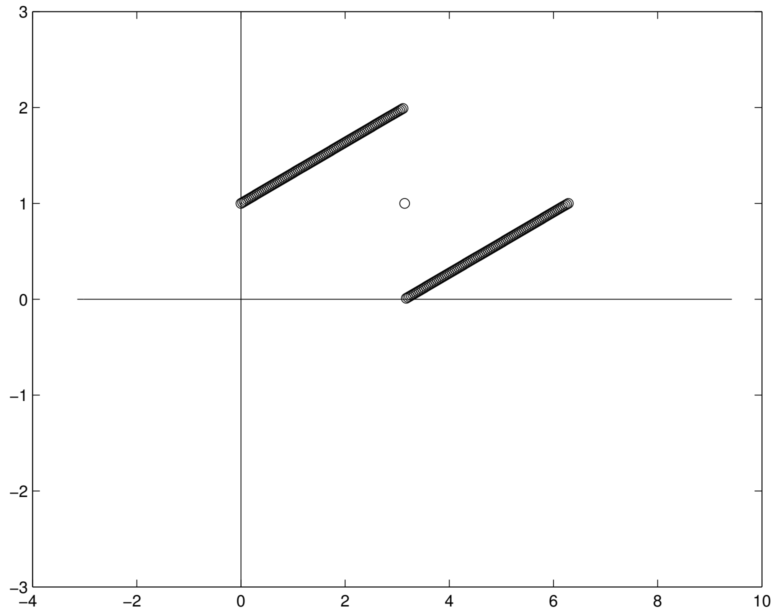

Figure 1:

4 $< 25pts >$

The function on Fig. 1, shown on the interval $(0, 2\pi)$, is described by the Fourier series

$$f(x) \equiv a_0 + \sum_{k=1}^{\infty} a_n \cos kx + b_n \sin kx \ .$$

1. Give a sketch on the interval $-\pi \leq x \leq 3\pi$.

2. Assume the same function is described over $0 \leq x \leq 2\pi$ by the Fourier sine series. Give a sketch over $-\pi \leq x \leq 3\pi$. Also give the form of the sine series (without computing the coefficients).

3. Repeat part (ii) for the cosine series.

Solution