Proceedings of the 5th International Conference on
Computation of Shell and Spatial Structures
June 1-4, 2005 Salzburg, Austria
E. Ramm, W.A. Wall, K.-U. Bletzinger, M. Bischoff (eds.)
www.iassiacm2005.de

Discrete constitutive equations for the formation, freezing and closing of leads in Arctic ice

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Abstract
Arctic ice motion is dominated by the initiation of cracks, opening of the cracks into leads, freezing of leads to form fresh ice, and the compaction of frozen leads to form ridges and keels. Since there are satellite data that indicate leads are often aligned with the direction of maximum compressive stress, a new decohesive model that allows for axial splitting is formulated. Preliminary results of lead predictions associated with kinematic data are given as is a possible constitutive approach for modeling lead closure and ridge formation.

1 Introduction

Sea ice plays a vital role in global atmospheric and ocean dynamics, as well as in determining ocean salinity; these, in turn, impact ocean currents, weather patterns and ecosystems throughout the globe. Mechanical deformation results in fracturing and ridging (opening and closing) of the ice cover. Cracks that open in the ice create areas of open water (leads) that significantly affect air-ice-ocean interaction. In winter, newly-opened cracks are the source of new ice growth, brine rejection to the ocean, and rapid heat transfer from the ocean to the atmosphere. Gaps in the ice cover that close cause ice to raft and pile up into pressure ridges, or to be forced down into subsurface keels, increasing the ice-atmosphere and ice-ocean drag. Enhancing the ability to forecast lead opening and closing will eventually impact our understanding of the Earth's weather, climate and ecology, and also impact our ability to navigate Arctic waters, especially if we can also predict the orientation, width and extent of leads.

The dominant feature of the Arctic ice sheet is the prevalence of leads (cracks) and ridges which are evidence of previous leads that have closed. Therefore any attempt to model Arctic ice must reflect the important kinematic features of displacement discontinuities and include constitutive equations suitable for the phenomena associated with the opening and closing of leads.

From a mechanics viewpoint, the problem is one of a membrane supported on a fluid. The membrane is weak in tension and strong in compression. In addition, the formation of the lead is typically a mixed mode involving both opening and shear. A discrete constitutive equation that appears to predict the correct features is described. A unique aspect of this project is that satellite data have been gathered that provide motions of a grid of material points over a significant part of the Arctic ice sheet. These data are used to kinematically drive the constitutive equation. The result is a partial verification of the constitutive equation in that there is a good correlation between predicted and observed data describing the orientation and magnitude of leads.

In addition, the constitutive issues related to the freezing and closing of leads are addressed. It is believed that such a model may have applications in other fields such as solidification of metals.

2 Decohesion model

Consider a potential surface of material failure with normal ***n*** and a unit tangent vector, ***t***, in the plane of the ice sheet. The components of the stress tensor become $\tau_n = \sigma_{nn}$, $\tau_t = \sigma_{nt}$ and σ_{tt}. The proposed criterion consists of two new features: (1) a modification of the Rankine criterion to allow for the possibility that a compressive stress component, σ_{tt}, may lower the resistance of the material to brittle failure, and (2) a combination of brittle and ductile aspects of failure within one criterion. The first feature is achieved by defining a brittle decohesion function as follows:

$$B_n = \frac{\tau_n}{\tau_{nf}} - f_n\left[\frac{<-\sigma_{tt}>^2}{f'^2_c} + 1\right] \quad where \quad <x> = \begin{matrix} x \text{ if } x \ge 0 \\ 0 \text{ if } x<0 \end{matrix} \tag{1}$$

in which f'_c denotes the failure stress in uniaxial compression. The ramp function is used to activate the normal component of stress σ_{tt} only if it is negative. The new criterion for brittle failure becomes one of searching over ***n*** for the maximum value of B_n and failure is indicated by $B_n = 0$. The result is analogous to the Rankine criterion in that the critical orientation is the direction of maximum principal stress, but the critical value of the normal component is reduced by the term involving the stress component σ_{tt}. The criterion allows for failure even if τ_n is negative. It follows that $-\infty < B_n \le 0$. The role of the parameter f_n, which has the initial value of one, will be discussed later.

The decohesion surface associated with the brittle criterion $B_n = 0$ for the initiation of failure and for plane stress is shown in Figure 1. The restriction is made that the normal to the surface of material failure must also lie in the same plane. The surface passes through the points of tensile failure, $\sigma_1 = \tau_{nf}$, and compressive failure, $\sigma_2 = -f'_c$, as desired. Also shown are arrows that indicate the normals to surfaces of material failure. These normals are always in the directions of maximum principal stress.

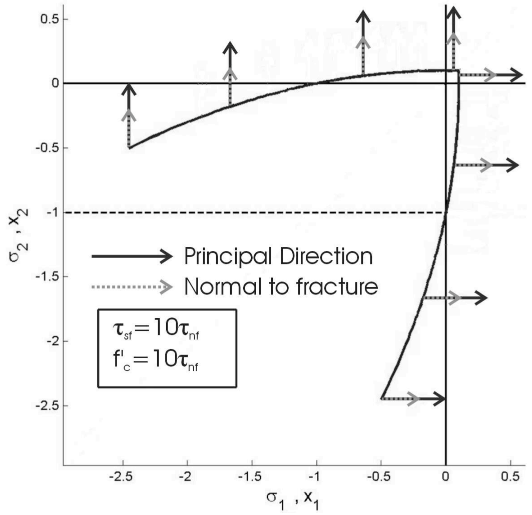


Figure 1: Proposed modified criterion for brittle failure.

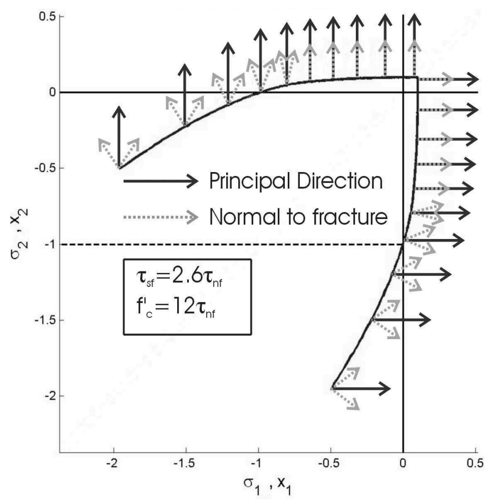


Figure 2: Combined model showing transition from brittle to ductile modes of failure.

Next, brittle and ductile aspects of failure are combined with the arbitrary choice of an exponential function by defining decohesion functions as follows:

$$F = \max_{n} F_n \qquad F_n = \frac{\tau_t^2}{\tau_{sm}^2} + e^{\kappa B_n} - 1 \tag{2}$$

The new material parameter, τ_{sm}, is the failure stress in shear when $\tau_n \to -\infty$. The parameter, κ, is derived from the criterion that the failure stress for the case of pure shear ($B_n = -1$) is τ_{sf}. If $\tau_{sf} = \infty$ the criterion reduces to the brittle case. If $\tau_{nf} = \infty$ and $f'_c = \infty$ then the criterion $F = 0$ reduces to that of Tresca. The surface of decohesion associated with the combined criterion is shown in Figure 2. Note that for failure with some shear, there are two possible surfaces of material failure. In numerical simulations, the surface is chosen for which the sign of the rotation associated with decohesion agrees with the sign of the rotation determined from the deformation field.

An associated evolution equation is used to determine increments in the components of discontinuity, $[u_n]$ and $[u_t]$. Although not necessary, the effective discontinuity is assumed to be the normal component $[u_n]$. The parameter f_n alluded to previously is a softening function of the form $f_n = 1 - ([u_n]/u_0)$ in which u_0 is a material parameter that introduces a length scale. If $[u_n] > u_0$, then $f_n = 0$ and the crack (lead) is fully open with no traction-carrying capacity on the free surfaces.

3 Applications to Arctic ice

The ultimate use of any constitutive model is an application to a boundary value problem. Here, one would hope to be able to predict the formation and closing of leads as a consequence of wind and ocean forces, Coriolis effects and boundary conditions associated with shore lines and open water. Prior to such an application, values of material parameters must be chosen. It is widely understood that the use of values from experimental data in the laboratory are inappropriate, probably because of the random array of thermal cracks and other variations that exist in Arctic ice. One possible method for estimating material parameters is to use measured kinematic data over a small segment of time to drive a section of ice and adjust parameters until predicted features match approximately those that are observed. The method is not perfect because the initial conditions of the ice are not known. Nevertheless, the approach is convenient and can provide reasonable estimates of material parameters.

To illustrate the procedure, consider kinematic data derived from high-resolution Synthetic Aperature Radar (SAR) imagery that are processed by the RADARSAT Geophysical Processor System (RGPS) developed at JPL (Kwok [1]). These data are presented in the form of motion of material points over a segment of time where the material points are defined as the nodes of a regular grid based on the initial observation. The locations of the same material points at the time of the second observation define a distorted mesh. An example of such data are given in Figure 3.

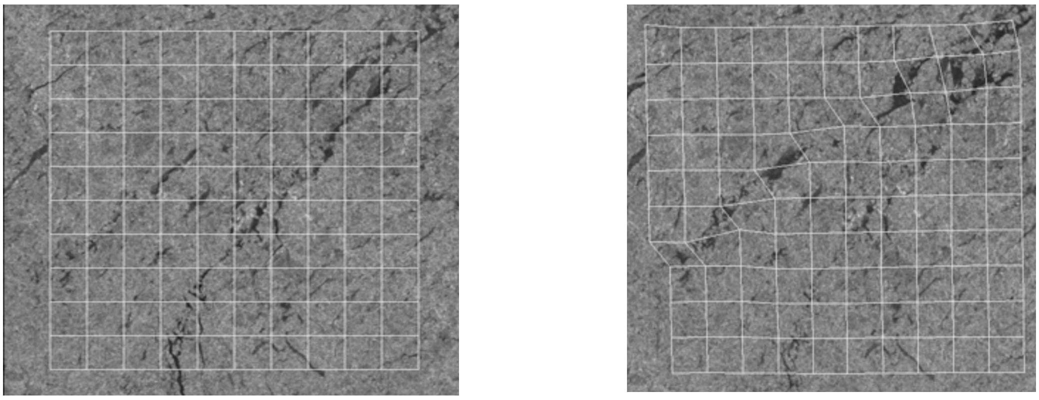
These grid motions were used to define strains at the centers of elements from which values of discontinuity in displacement were determined to define the possibility of crack and, ultimately, lead formation. The results for the given set of material parameters are shown in Fig. 4. Various choices for the shear-strength parameter, τ_{sf}, provide changes in predicted crack orientations and openings even though the deformations of the elements are identical from one case to the next. It is suggested that by comparing these observed and predicted data, optimal values of material parameters can be chosen for use in obtaining solutions to boundary value problems.

4 Preliminary Model for Freezing and Ridging

Once a crack initiates, the strength in compression is also reduced. Such a feature is accomodated by introducing an additional decohesion function, F_n^c, with its own softening function, f_n^c, as follows:

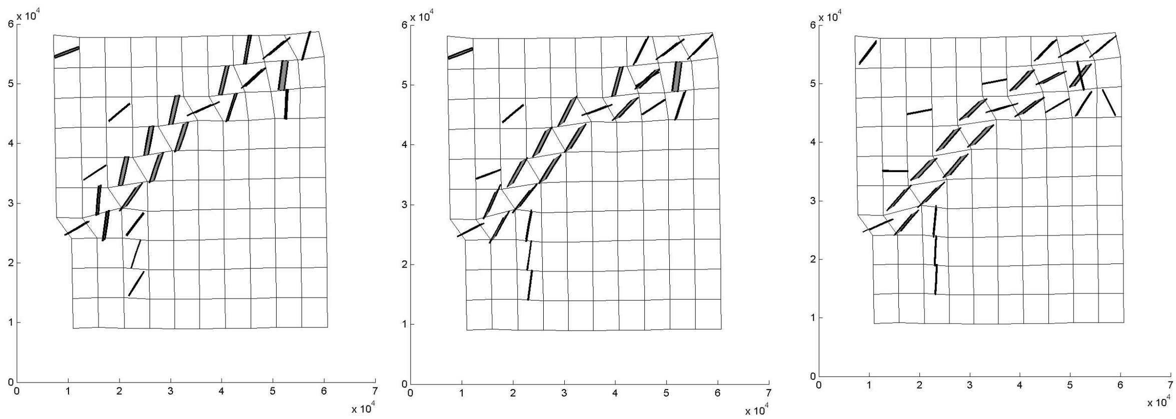
$$F_n^c = -\tau_n - f_n^c \tag{3}$$

In order for F_n^c to be positive or zero, τ_n must be negative. Now evolution equations are constructed for f_n^c as follows: (1) during the creation of the crack, f_n^c must decrease to the limiting value of zero as a lead is formed, (2) for freezing, f_n^c must increase, and (3) for lead closure, f_n^c must also increase. There are experimental data that provide general guidance as to what form these evolution equations must take and is the topic of current research on the development of the model.



(a) Initial configuration. (b) Final configuration.

Figure 3: RGPS data for motion over a time span of 18.5 hrs. for a representative region of 50 km x 50 km.



(a) $\tau_{sf} = 1.4\tau_{nf}$ (b) $\tau_{sf} = \tau_{nf}$ (c) $\tau_{sf} = 0.6\tau_{nf}$

Figure 4: Predicted crack formation for $\tau_{nf} = 25kPa, f'_c = 10\tau_{nf}$ and $\tau_{sm} = 6\tau_{sf}$.

5 Conclusions

In order to predict the initiation and evolution of leads in Arctic ice a discrete constitutive equation for the evolution of displacement discontinuity is proposed. The unique features of the model are: (1) the prediction of axial splitting, (2) the possibility of predicting different modes of failure for different paths in stress or strain, (3) the prediction of the orientation of the surface of material failure, (4) the ability to handle multiple cracks at a point, a necessary feature for predicting crack branching, (5) the capability of presetting surfaces of weakness, and (6) the ability to recover strength due to freezing and compaction. Each of these features has been implemented numerically and preliminary analyses have been performed on segments of Arctic ice using the material point method (Schreyer et al. [2]).

References

[1] Kwok, R.(1998). The RADARSAT Geophysical Processor System, in *Analysis of SAR data of the Polar Oceans: Recent Advances*, Tsatsoulis, C. and R. Kwok, Eds., Springer-Verlag, pp. 235-257.

[2] Schreyer, H.L., Sulsky, D.L., and Zhou, S.-J.(2002). Modeling delamination as a strong discontinuity with the material point method, *Computer Methods in Applied Mechanics and Engineering*, 191(23-24):2483-2508.