A DECOHESIVE REPRESENTATION OF ICE FAILURE

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ABSTRACT

A discrete constitutive equation is described for predicting the initiation and evolution of leads in Arctic ice. A key feature of the new model is the ability to predict axial splitting under uniaxial compression with a suitable choice of material parameters. Other unique aspects of the model are the capability to handle multiple cracks at a point, to predict modes of failure according to the state of stress, and to build in pre-existing planes of weakness that may be due, for example, to frozen leads. A method is suggested for estimating values of material parameters for *in situ* ice by utilizing satellite data describing the motion of Arctic ice.

INTRODUCTION

Many simulations of failure in Arctic ice, as reflected by the formation of leads, are based on the utilization of a continuum constitutive equation. Such an approach requires an additional technique to infer lead initiation and development. One is to monitor the sign of the divergence of the velocity field with a positive sign indicating the development of open water. Another is to define a parameter that indicates when well-posedness of the governing partial differential equations is lost. Alternatively, one can use a fracture criterion such as a critical value of maximum principal stress. None provides detailed information on the nature and development of the lead.

This presentation describes a new cohesive crack model (also called a decohesive or discrete constitutive equation) that specifically indicates when a lead initiates, provides the orientation of the lead, and includes evolution equations that describe how a lead develops so that the relative tangential and normal motion of one side of the lead with respect to the other can be obtained.

Decohesive models are extensively invoked in numerical simulations such as the example given in Schreyer et al. (2002) but surprisingly little effort has been expended on formulating versions appropriate for any specific material. For example, a key requirement necessary for geological materials, including ice, is a feature that provides for axial splitting. In addition there is a need to transition from axial splitting to a mixed mode with a change of stress if the complete failure process is to be properly represented. These aspects are included in the formulation described here.

As part of the process of constitutive equation verification, observed ice motion over a small region of ice is used. These kinematic data sets are derived from high-resolution Synthetic Aperature Radar (SAR) imagery that is processed by the RADARSAT Geophysical Processor System (RGPS) developed at JPL (Kwok, 1998). These data sets are used to provide an estimate for values of material constants. Also, the correct overall features of lead directions and openings should be predicted if there is to be any hope for predicting essential features of lead formation over larger domains and periods of time.

CONVENTIONAL MODELS

Any ice sheet is essentially in a state of plane stress and the normal to the failure surface typically lies within the plane. Therefore, it is appropriate to consider only a two-dimensional theory. Let the components of the stress tensor, $\boldsymbol{\sigma}$, be σ_{11}, σ_{22} and σ_{12} with respect to Cartesian coordinates x_1 and x_2. Introduce a local basis, **n** and **t**, where **n** represents the unit normal to a potential surface of failure and **t** is a unit vector tangent to the surface. Corresponding components of stress are $\tau_n = \sigma_{nn}$, $\tau_t = \sigma_{nt}$ and σ_{tt} where τ_n and τ_t are components of the traction vector. For a given orientation, **n**, is assumed that a decohesion function $F_n(\boldsymbol{\sigma},\mathbf{n})$ can be defined such that $F_n < 0$ implies no failure, $F_n > 0$ is not allowed and $F_n = 0$ denotes evolution of failure in the form of cracking for that orientation. With the choice of sign indicated above, a general decohesion function defined as follows provides the critical case:

$$(1) \qquad F = \max_{\boldsymbol{n}} F_n$$

Typically, the classical models fall in a category where F_n is chosen to be a function of the traction, $\boldsymbol{\tau}$, rather than the stress. Examples are:

(2) $F_n = \frac{\tau_n}{\tau_{nf}} - 1 \qquad F_n = \frac{|\tau_t|}{\tau_{sf}} - 1 \qquad F_n = \frac{|\tau_t|}{\tau_{sf}} + C\tau_n - 1$

associated with the names of Rankine, Tresca and Mohr-Coulomb, respectively. Material parameters consist of the tensile or normal failure stress, τ_{nf}, the failure stress in pure shear, τ_{sf}, and a Coulomb factor, *C*.

PROPOSED MODEL

The proposed criterion consists of two new features: (1) a modification of the Rankine criterion to allow for the possibility that a compressive stress component, σ_{tt}, may lower the resistance of the material to brittle failure, and (2) a combination of brittle and ductile aspects of failure within one criterion. The first feature is achieved by defining a brittle decohesion function as follows:

(3) $B_n = \frac{\tau_n}{\tau_{nf}} - [\frac{<-\sigma_{tt}>^2}{f'^2_c} + 1] \quad where \quad <x> = \begin{matrix} x \text{ if } x \geq 0 \\ 0 \text{ if } x<0 \end{matrix}$

in which f'_c denotes the failure stress in uniaxial compression. The ramp function is used to activate the normal component of stress σ_{tt} only if it is negative. The new criterion for brittle failure becomes one of searching over $\boldsymbol{n}$ for the maximum value of B_n and failure is indicated by $B_n = 0$. The result is analogous to the Rankine criterion in that the critical orientation is the direction of maximum principal stress, but the critical value of the normal component is reduced by the term involving the second principal stress σ_{tt}. The criterion allows for failure even if τ_n is negative. It follows that $-\infty < B_n \leq 0$.

Ductile materials typically fail by rupture based on a shear criterion. Although ice is not generally considered to be ductile, it is very possible that if shear is part of the failure mechanism, considerably more energy will be dissipated than for a normal (brittle) mode of failure. Therefore, shear failure will be called ductile and a mixed mode failure can be thought of as displaying both brittle (Mode I) and ductile (Mode II) features. Brittle and ductile aspects of failure are incorporated within the model through the arbitrary choice of an exponential function by defining decohesion functions as follows:

(4) $F = \max_{\boldsymbol{n}} F_n \qquad F_n = \frac{\tau_t^2}{\tau_{sm}^2} + e^{\kappa B_n} - 1$

The new material parameter, τ_{sm}, is the failure stress in shear when $\tau_n \rightarrow -\infty$. The parameter, κ, is derived from the criterion that the failure stress in shear is τ_{sf} for the case of pure shear ($B_n = -1$). If $\tau_{nf} = \infty$ and $f'_c = \infty$ then the criterion $F = 0$ reduces to that of Tresca.

The various surfaces for the initiation of failure in terms of principal stresses for plane stress are shown in Figs. 1 and 2. Because of symmetry, only the domain of $\sigma_1 \geq \sigma_2$ is used to describe the classical surfaces of Rankine and Tresca in Fig. 1. Also shown are arrows that indicate the normals to surfaces of material failure. Note that for shear failure, there are two possible directions, one orthogonal to the other. Next, the surfaces of decohesion associated with the proposed criterion are shown in Fig. 2. The first example is the brittle case obtained by setting $\tau_{sm} = \infty$ for which $F_n = 0$ implies that $B_n = 0$. The surface passes through the points of tensile failure, $\sigma_1 = \tau_{nf}$, and compressive failure, $\sigma_2 = -f'_c$, as desired. Furthermore, the normal to the failure surface is always in the direction of maximum principal stress.

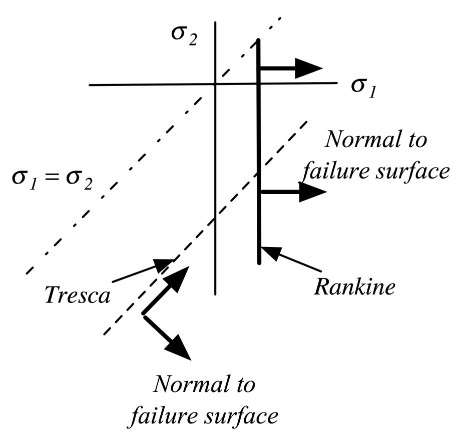


Fig. 1. Failure surfaces of Rankine and Tresca.

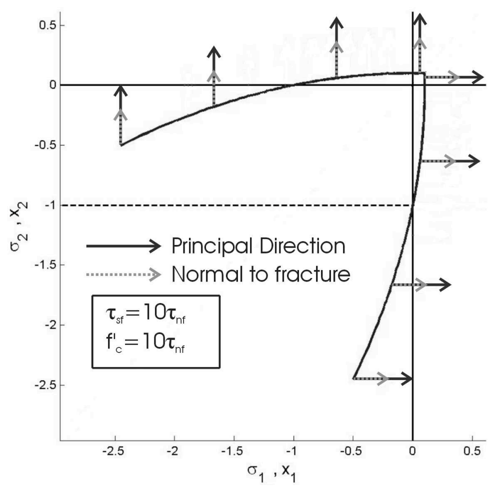


Fig. 2. Proposed modification to the Rankine criterion for brittle failure

The next case shown in Fig. 3 is that for a reduced and more physically reasonable value for maximum shear strength of $\tau_{sm} = 4\tau_{sf}$. Again, for uniaxial tension, $\sigma_1 = \tau_{nf}$, and the direction of failure remains that of maximum principal stress. However, as the second principal stress is reduced, a transition occurs in which two normals appear. These directions make equal angles with the direction of maximum principal stress. Now, failure under uniaxial compression does not appear as axial splitting; instead, the orientation of a surface of failure exists as one of two possibilities. With an even further decrease in σ_2 the criterion has transitioned smoothly to that of Tresca. The result is that for some states of stress, failure is brittle whereas for "biaxial compressive" states the failure is ductile. Both types are observed (Schulson, 2004). Even under uniaxial compression brittle failure is observed sometimes, and under different temperatures or strain rates, ductile failure holds (Sammonds et al., 1998; Schulson, 2001). The current model can accommodate either situation through the selection of a suitable value for τ_{sm}. With the ductile feature, note that to make the failure stress in compression ten times that in tension, it is necessary to choose a larger value of f'_c.

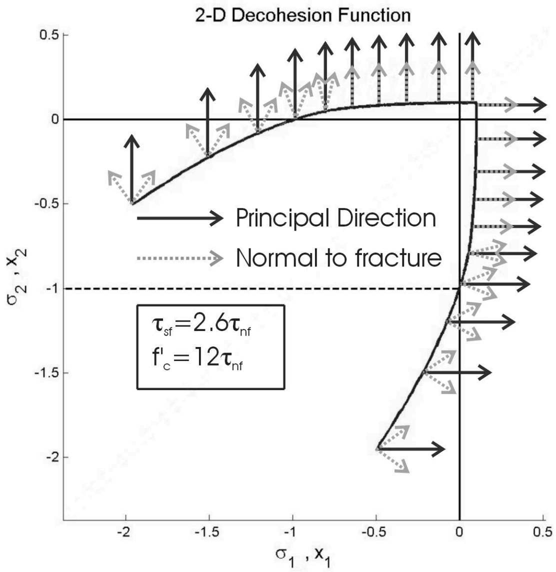


Fig. 3. Transition from brittle to ductile failure.

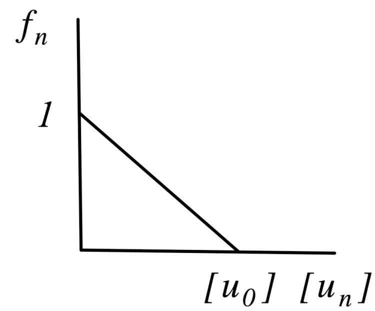


Fig. 4. Softening as a function of displacement discontinuity

To allow for the description of failure as a continuous process of decohesion whereby the traction-carrying capacity is reduced as the displacement discontinuity of a crack evolves, a softening term, f_n, is introduced as a function of effective discontinuity. For the sake of simplicity, the effective discontinuity is chosen to be the normal component of discontinuity, $[u_n]$, and a linearly decaying relation is assumed as shown in Fig. 4. For $[u_n] > [u_0]$, the traction carrying capacity is zero. The notation f_n, is used to emphasize the point that a softening function exists for every orientation, ***n***. Softening is incorporated into the model by assuming the brittle decohesion function is modified to be the following:

$$B_n = \frac{\tau_n}{\tau_{nf}} - f_n\left[\frac{<-\sigma_{tt}>^2}{f'^2_c} + 1\right] \tag{5}$$

The combined decohesion function remains the form given in (4). As indicated previously, for a given stress path, the direction that maximizes the decohesion function provides the direction ***n***. When cracking evolves, the value of f_n decreases for that orientation, and eventually a full crack exists as identified with the condition $f_n = 0$.

EVOLUTION EQUATIONS

To describe the evolution of crack formation, an associated relation is assumed and given as follows for the two components of displacement discontinuity:

$$(6) \qquad [\dot{u}_n] = \dot{\omega}\frac{\partial F}{\partial \tau_n} \qquad\qquad [\dot{u}_t] = \dot{\omega}\frac{\partial F}{\partial \tau_t}$$

For numerical applications to a square element of side h, components of a decohesion strain tensor are defined to be

$$(7) \qquad \dot{e}_{nn}^{dc} = [\dot{u}_n] / h_n \qquad\qquad \dot{e}_{nt}^{dc} = [\dot{u}_t] / 2h_n \qquad\qquad \dot{e}_{tt}^{dc} = 0$$

with $h_n = h^2 / h_t$ and h_t defined in Fig. 5. The total strain is considered to be the sum of elastic and decohesive strains so that conventional plasticity-type algorithms can be used to handle decohesion.

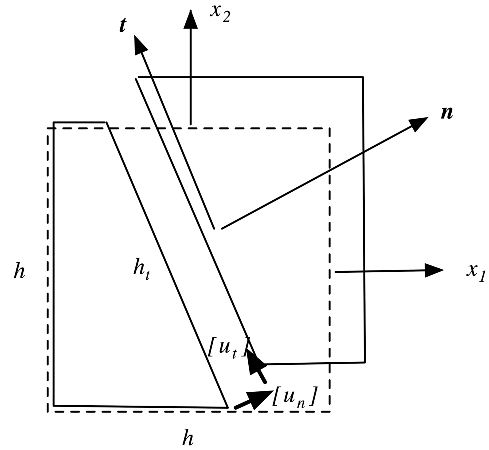


Fig. 5. Notation for decohesion components within an element.

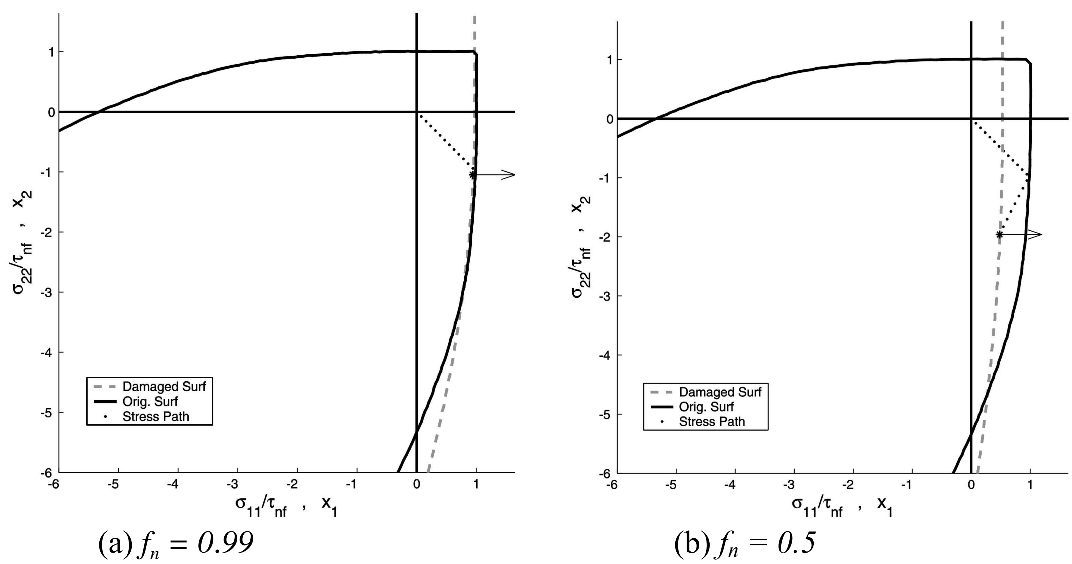


Fig. 6. Evolution of decohesion surfaces for a shear path in strain space.

An example is given in Fig. 6 for a shear path in strain, $e_1 = -e_2$. The principal stresses follow a similar path until the failure surface is reached with ***n*** the direction of maximum principal stress. Now two decohesive surfaces exist, one associated with the normal to the surface of material failure and one for all other directions. The two cases shown in Fig. 6 represent the situations of initial failure, $f_n = 0.99$, and of partial failure, $f_n = 0.5$. Although the material is initially isotropic, once failure initiates, the model is anisotropic and even though the path remains one of pure shear in strain, the stress path is not one of pure shear. The importance of anisotropy in representing sea ice has previously been strongly emphasized by Coon et al. (1998).

As an additional example, suppose that a previous lead has frozen. It is possible to build in a weakness for a particular direction as indicated in Fig. 7 for which full strength is indicated for all directions except $\theta=\theta^*$. The weakness can be preset by assuming a value for f_n and constructing the corresponding decohesion surface shown in Fig. 8. To determine if a previous fault or an existing partial crack is activated, the value for F_n is determined for that orientation. If $F_n > 0$, then the crack displacement must be increased and f_n decreased to the point where $F_n = 0$. Then to check for the initiation of new cracks, f_n is set to unity and F_n evaluated for all **n**. If $F_n > 0$ anywhere, then a new crack must be initiated for that orientation. Typically, new cracks do not form with orientations close to an existing one because the stress field is significantly altered in the presence of a partial crack.

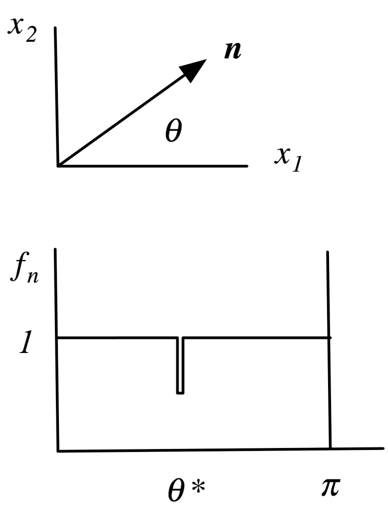


Fig. 7. Example showing that f_n reflects weakness in the direction $\theta=\theta^*$.

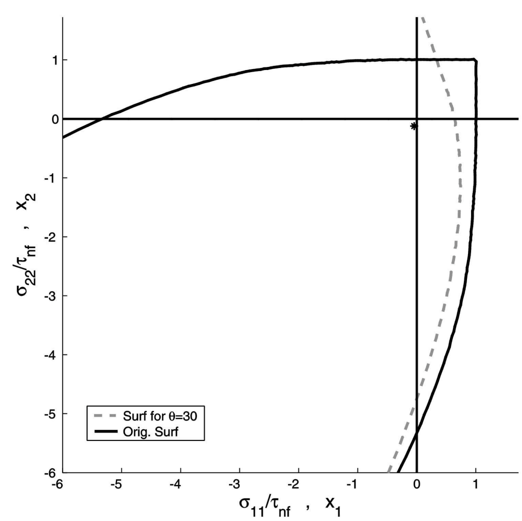


Fig. 8. Additional decohesion surface associated with $\theta=\theta^*=30^0$.

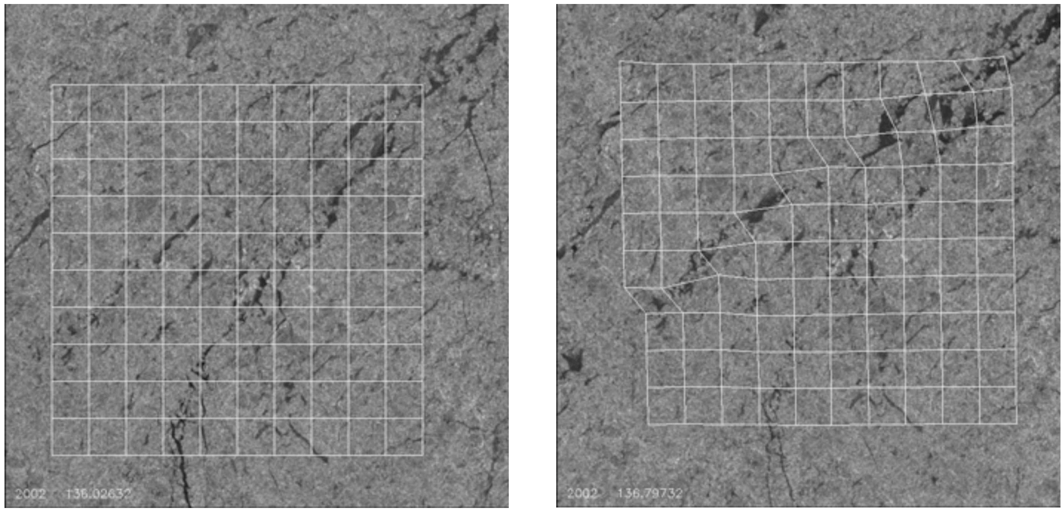
APPLICATION TO ARCTIC ICE

The ultimate use of any constitutive model is an application to a boundary value problem. Here, one would hope to be able to predict the formation and closing of leads as a consequence of wind and ocean forces, Coriolis effects and boundary conditions associated with shore lines and open water. Prior to such an application, values of material parameters must be chosen. It is widely understood that the use of values from experimental data in the laboratory are inappropriate, probably because of the random array of thermal cracks and other variations that exist in Arctic ice. One possible method for estimating material parameters is to use measured kinematic data over a small segment of time to drive a section of ice and adjust parameters until predicted features match approximately those that are observed. The method is not perfect because the initial conditions of the ice are not known. Nevertheless, the approach is convenient and might provide reasonable estimates of material parameters.

To illustrate the procedure, consider kinematic data derived from high-resolution Synthetic Aperature Radar (SAR) imagery that are processed by the RADARSAT Geophysical Processor System (RGPS) developed at JPL (Kwok, 1998). These data are presented in the form of motion of material points over a segment of time where the material points are defined as the nodes of a regular grid based on the initial observation. The locations of the same material points at the time of the second observation define a distorted mesh. An example of such data are given in Fig. 9, defined arbitrarily here as Region 3.

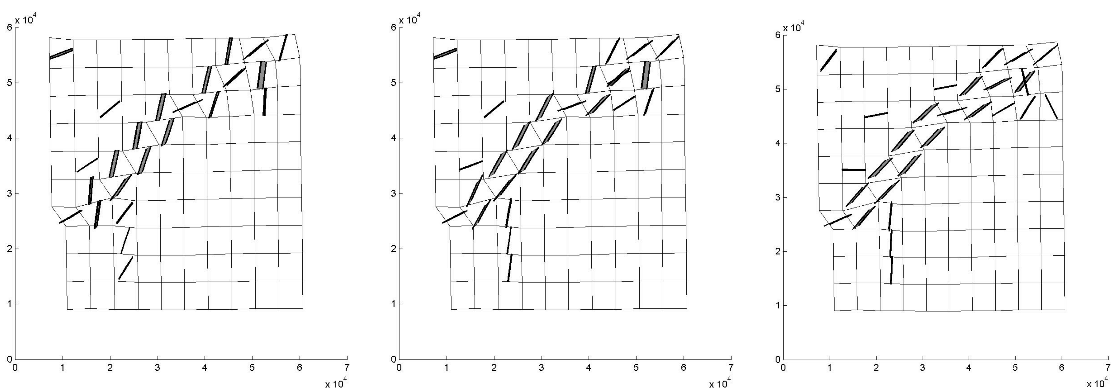
These grid motions were used to define strains at the centers of elements from which values of discontinuity in displacement were determined to define the possibility of crack and, ultimately, lead formation. The results for the given set of material parameters are shown in Fig. 10. Various choices for the shear-strength parameter, τ_{sf}, provide changes in predicted crack orientations and openings even though the deformations of the elements are identical from one case to the next. The value for the tensile strength parameter, $\tau_{nf} = 25kPa$, was chosen based on a similar iterative procedure and is considerably smaller than values associated with laboratory ice. The need for small values of strength has been noted by researchers involved with large-scale predictions of ice motion. It has been suggested that an even further reduction by an order of magnitude is necessary. Such a reduction would imply, in effect, that no tensile strength exists. An alternative interpretation to a possible situation that reflects low tensile stresses is that leads exist somewhere within the domain, and with the proposed model, a lead would be brought explicitly into the analysis by adjusting the decohesion surface to reflect zero strength in the normal direction for material points along the lead.

To provide an objective measure of how one value of a material parameter should be selected over another, an indicator must be developed to show the error between displacement discontinuities predicted through the use of the constitutive equation and those indicated by the observed data. At this stage, it is not clear that the accuracy implicit with both the theoretical and the measured data warrant such a degree of refinement. Instead, the only claims that are made are that the model provides features not present in existing models and that the analysis based on kinematic data result in values of model parameters in line with those of existing models, when such comparisons are appropriate.



(a) Initial configuration. (b) Final configuration.

Fig. 9. RGPS data for "Region 3" for motion over a time span of 18.5 hrs.



(a) $\tau_{sf} = 1.4\tau_{nf}$ (b) $\tau_{sf} = \tau_{nf}$ (c) $\tau_{sf} = 0.6\tau_{nf}$

Fig. 10. Predicted crack formation for $\tau_{nf} = 25kPa, f'_c = 10\tau_{nf}$ and $\tau_{sm} = 6\tau_{sf}$.

CONCLUSIONS

In order to predict the initiation and evolution of leads in Arctic ice a discrete constitutive equation for the evolution of displacement discontinuity is proposed. The unique features of the model are (1) the prediction of axial splitting, (2) the possibility of predicting different modes of failure for different paths in stress or strain, (3) the prediction of the orientation of the surface of material failure, (4) the ability to handle multiple cracks at a point, and (5) the capability of presetting surfaces of weakness. Each of these features has been implemented numerically and preliminary analyses have been performed on segments of Arctic ice. Alternative constitutive equations for lead closure with ridging are currently being formulated.

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