In[2]:=
```
(* Following Section 3.12 in  "The fuzzy space construction kit"
 by Andreas Sykora, Calculating the index for r=1 *)
```

In[3]:=
```
r = 1;
```

In[4]:=
```
n = 6;
```

In[5]:=
```
V = {{8/10, 1, 1, 0, 0, 0}, {0, 0, 0 , 1, 0, 0}, {0, 0, 16/10, r, 1, 0},
   {0, 0, 0, 8/10, 0, 1}, {0, 0, 0, 0, 24/10, 1}, {0, 0, 0, 0, 0, 16/10}};
```

In[6]:=
```
MatrixForm[V]
```

Out[6]//MatrixForm=

$$\begin{pmatrix} \frac{4}{5} & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & \frac{8}{5} & 1 & 1 & 0 \\ 0 & 0 & 0 & \frac{4}{5} & 0 & 1 \\ 0 & 0 & 0 & 0 & \frac{12}{5} & 1 \\ 0 & 0 & 0 & 0 & 0 & \frac{8}{5} \end{pmatrix}$$

In[7]:=
```
Z = (1/10) * {{0, 0, 0, 0, 0, 0}, {0, 13, 0, 0, 0, 0}, {0, 0, 13, 0, 0, 0},
   {0, 0, 0, 26, 0, 0}, {0, 0, 0, 0, 26, 0}, {0, 0, 0, 0, 0, 39}};
```

In[8]:=
```
MatrixForm[Z]
```

Out[8]//MatrixForm=

$$\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{13}{10} & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{13}{10} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{13}{5} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{13}{5} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{39}{10} \end{pmatrix}$$

In[9]:=
```
X = (1/2) * ConjugateTranspose[V] + (1/2) * V;
```

In[10]:=
```
MatrixForm[X]
```

Out[10]//MatrixForm=

$$\begin{pmatrix} \frac{4}{5} & \frac{1}{2} & \frac{1}{2} & 0 & 0 & 0 \\ \frac{1}{2} & 0 & 0 & \frac{1}{2} & 0 & 0 \\ \frac{1}{2} & 0 & \frac{8}{5} & \frac{1}{2} & \frac{1}{2} & 0 \\ 0 & \frac{1}{2} & \frac{1}{2} & \frac{4}{5} & 0 & \frac{1}{2} \\ 0 & 0 & \frac{1}{2} & 0 & \frac{12}{5} & \frac{1}{2} \\ 0 & 0 & 0 & \frac{1}{2} & \frac{1}{2} & \frac{8}{5} \end{pmatrix}$$

In[11]:=
```
Y = (i/2) * ConjugateTranspose[V] - (i/2) * V;
```

In[12]:=
```
0
```

Out[12]=
```
0
```

In[13]:=
```
MatrixForm[Y]
```

Out[13]//MatrixForm=

$$\begin{pmatrix} 0 & -\frac{i}{2} & -\frac{i}{2} & 0 & 0 & 0 \\ \frac{i}{2} & 0 & 0 & -\frac{i}{2} & 0 & 0 \\ \frac{i}{2} & 0 & 0 & -\frac{i}{2} & -\frac{i}{2} & 0 \\ 0 & \frac{i}{2} & \frac{i}{2} & 0 & 0 & -\frac{i}{2} \\ 0 & 0 & \frac{i}{2} & 0 & 0 & -\frac{i}{2} \\ 0 & 0 & 0 & \frac{i}{2} & \frac{i}{2} & 0 \end{pmatrix}$$

In[14]:=
```
sigma1 = {{0, 1}, {1, 0}};
```

In[15]:=
```
sigma2 = {{0, -I}, {I, 0}};
```

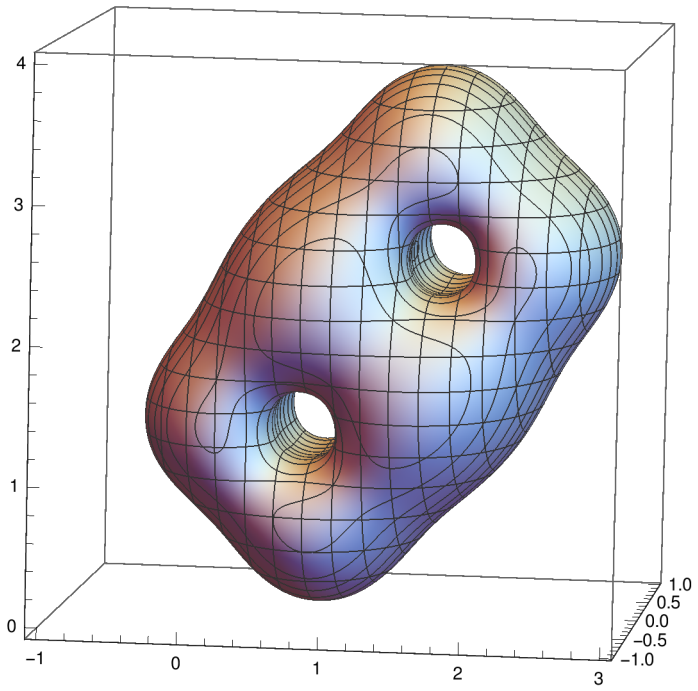
In[16]:=
```
sigma3 = {{1, 0}, {0, -1}};
```

In[17]:=
```
loclzr = KroneckerProduct[sigma1, X - x * IdentityMatrix[n]] +
   KroneckerProduct[sigma2, Y - y * IdentityMatrix[n]] +
   KroneckerProduct[sigma3, Z - z * IdentityMatrix[n]] ;
```

In[18]:=
```
charpoly = Det[loclzr];
```

In[19]:=
```
ContourPlot3D[charpoly == 0, {x, -1, 3}, {y, -1, 1}, {z, 0, 4},
 Contours → {{1, LightBlue}}, PlotPoints → 100, ViewPoint → {2, -18, 2}]
```

Out[19]=



In[26]:=
```
Eigenvalues[N[ReplaceAll[loclzr, {x → 2, y → 0, z → 1 / 4}]]]
(*This point has index -1 *)
```

Out[26]= {4.02198, −3.67852, 3.33598, −2.95729, 2.66918,
−2.57137, 2.43667, −2.39928, 1.6046, −1.51907, −1.3799, 0.437004}