```mathematica
In[109]:= n = 6;

In[110]:= sigmax = {{0, 1}, {1, 0}};

In[111]:= sigmay = {{0, -i}, {i, 0}};

In[112]:= sigmaz = {{1, 0}, {0, -1}};

In[113]:= I2 = IdentityMatrix[2];

In[114]:= u = DiagonalMatrix[ConstantArray[1, n - 1], 1] +
        DiagonalMatrix[ConstantArray[1, 1], -n + 1];

In[115]:= v = DiagonalMatrix[Exp[2 Pi I / n]^Range[n]];

In[116]:= AA = (1 / 2) * (ConjugateTranspose[u] + u);

In[117]:= BB = (i / 2) * (ConjugateTranspose[u] - u);

In[118]:= CC = (1 / 2) * (ConjugateTranspose[v] + v);

In[119]:= DD = (i / 2) * (ConjugateTranspose[v] - v);

In[120]:= loclzr = KroneckerProduct[i * sigmax, AA - w * IdentityMatrix[n]] +
        KroneckerProduct[i * sigmay, BB - x * IdentityMatrix[n]] +
        KroneckerProduct[i * sigmaz, CC - y * IdentityMatrix[n]] +
        KroneckerProduct[I2, DD - z * IdentityMatrix[n]];

In[121]:= charpoly = Det[loclzr];

In[122]:= impoly = FullSimplify[Im[charpoly],
        {Element[w, Reals], Element[x, Reals], Element[y, Reals], Element[z, Reals]}]
```

$$\text{Out[122]=} \quad \frac{3}{2} \sqrt{3} \, \left(w^2 + x^2 - y^2 - z^2\right) \, \left(2 + w^2 + x^2 + y^2 + z^2\right)$$
$$\left(9 + 6 \, x^2 + 6 \, y^2 + 6 \, z^2 + 2 \, \left(w^4 + \left(x^2 + y^2 + z^2\right)^2 + w^2 \, \left(3 + 2 \, x^2 + 2 \, y^2 + 2 \, z^2\right)\right)\right)$$

```mathematica
In[123]:= therest = FullSimplify[ impoly / (w^2 + x^2 - y^2 - z^2)]
```

$$\text{Out[123]=} \quad \frac{3}{2} \sqrt{3} \, \left(2 + w^2 + x^2 + y^2 + z^2\right)$$
$$\left(9 + 6 \, x^2 + 6 \, y^2 + 6 \, z^2 + 2 \, \left(w^4 + \left(x^2 + y^2 + z^2\right)^2 + w^2 \, \left(3 + 2 \, x^2 + 2 \, y^2 + 2 \, z^2\right)\right)\right)$$

```mathematica
In[124]:= realpoly = FullSimplify[Re[charpoly], {w^2 + x^2 == y^2 + z^2,
        Element[w, Reals], Element[x, Reals], Element[y, Reals], Element[z, Reals]}];

In[125]:= realpoly = ReplaceAll[realpoly, {x → Sqrt[-w^2 + y^2 + z^2]}];

In[126]:= realpoly =
        FullSimplify[realpoly, {Element[w, Reals], Element[y, Reals], Element[z, Reals]}]
```

$$\text{Out[126]=} \quad -64 \, w^6 + 64 \, y^{12} + 96 \, w^4 \, \left(y^2 + z^2\right) - 36 \, w^2 \, \left(y^2 + z^2\right)^2 + 192 \, y^{10} \, \left(1 + 2 \, z^2\right) + 240 \, y^8 \, \left(1 + 2 \, z^2\right)^2 +$$
$$\left(-3 - z^2 + 8 \, z^4 + 8 \, z^6\right) \, \left(9 + 15 \, z^2 + 16 \, z^4 + 8 \, z^6\right) + 6 \, y^2 \, \left(-9 + 3 \, z^2 + 70 \, z^4 + 160 \, z^6 + 160 \, z^8 + 64 \, z^{10}\right) +$$
$$4 \, y^6 \, \left(37 + 80 \, z^2 \, \left(3 + 6 \, z^2 + 4 \, z^4\right)\right) + 3 \, y^4 \, \left(3 + 160 \, \left(z^2 + 3 \, z^4 + 4 \, z^6 + 2 \, z^8\right)\right)$$

In[127]:= `altpoly = ReplaceAll[realpoly, {w → r * Cos[th], y → r * Cos[phi], z → r * Sin[phi]}];`

In[128]:= `altpoly = FullSimplify[altpoly, {r > 0, Element[th, Reals], Element[phi, Reals]}];`

In[145]:= `Collect[ExpandAll[altpoly], r]`

Out[145]= $-27 - 54 r^2 + 9 r^4 + 240 r^8 + 192 r^{10} + 64 r^{12} + r^6 \left(148 - 2 \cos[6 \, phi] - 2 \cos[6 \, th]\right)$

In[146]:= `estLeft = -27 + 9 (6 / 10)^4 + 240 (6 / 10)^8 + 192 (6 / 10)^10 + 64 (6 / 10)^12`

Out[146]= $-\dfrac{5\,005\,433\,826}{244\,140\,625}$

In[147]:= `N[estLeft]`

Out[147]= -20.5023

In[130]:= `Collect[Expand[D[altpoly, r]], r]`

Out[130]= $-108 r + 36 r^3 + 1920 r^7 + 1920 r^9 + 768 r^{11} + r^5 \left(888 - 12 \cos[6 \, phi] - 12 \cos[6 \, th]\right)$

In[148]:= `estRight =`
`  -108 × 1 + 36 (6 / 10)^3 + 1920 (6 / 10)^7 + 1920 (6 / 10)^9 + 768 (6 / 10)^11 + (6 / 10)^5 (888 - 24)`

Out[148]= $\dfrac{2\,091\,982\,896}{48\,828\,125}$

In[149]:= `N[estRight]`

Out[149]= 42.8438

In[131]:= `ContourPlot3D[realpoly == 0, {w, -1, 1}, {y, -1, 1},`
`  {z, -1, 1}, RegionFunction → Function[{w, y, z}, 0 < y^2 + z^2 - w^2],`
`  ColorFunction → Function[{w, y, z},`
`    ColorData["TemperatureMap"][0.5 + 0.5 Sqrt[y^2 + z^2 - w^2]]],`
`  ColorFunctionScaling → False, ViewPoint → {-5, -8, 4},`
`  BoxRatios → Automatic, PlotPoints → 25, MaxRecursion → 3]`
`ContourPlot3D[realpoly == 0, {w, -1, 1}, {y, -1, 1},`
`  {z, -1, 1}, RegionFunction → Function[{w, y, z}, 0 < y^2 + z^2 - w^2],`
`  ColorFunction → Function[{w, y, z},`
`    ColorData["TemperatureMap"][0.5 - 0.5 * Sqrt[y^2 + z^2 - w^2]]],`
`  ColorFunctionScaling → False, ViewPoint → {-5, -8, 4},`
`  BoxRatios → Automatic, PlotPoints → 25, MaxRecursion → 3]`

Out[131]=

Out[132]=