In[276]:=
```
n = 4;
```

In[277]:=
```
sigmax = {{0, 1}, {1, 0}};
```

In[278]:=
```
sigmay = {{0, -i}, {i, 0}};
```

In[279]:=
```
sigmaz = {{1, 0}, {0, -1}};
```

In[280]:=
```
I2 = IdentityMatrix[2];
```

In[281]:=
```
u = DiagonalMatrix[ConstantArray[1, n - 1], 1] +
    DiagonalMatrix[ConstantArray[1, 1], -n + 1];
```

In[282]:=
```
v = DiagonalMatrix[Exp[2 Pi I / n] ^ Range[n]];
```

In[283]:=
```
AA = (1 / 2) * (ConjugateTranspose[u] + u);
```

In[284]:=
```
BB = (i / 2) * (ConjugateTranspose[u] - u);
```

In[285]:=
```
CC = (1 / 2) * (ConjugateTranspose[v] + v);
```

In[286]:=
```
DD = (i / 2) * (ConjugateTranspose[v] - v);
```

In[287]:=
```
loclzr = KroneckerProduct[i * sigmax, AA - w * IdentityMatrix[n]] +
    KroneckerProduct[i * sigmay, BB - x * IdentityMatrix[n]] +
    KroneckerProduct[i * sigmaz, CC - y * IdentityMatrix[n]] +
    KroneckerProduct[I2, DD - z * IdentityMatrix[n]];
```

In[288]:=
```
charpoly = Det[loclzr];
```

In[289]:=
```
impoly = FullSimplify[Im[charpoly],
  {Element[w, Reals], Element[x, Reals], Element[y, Reals], Element[z, Reals]}]
```

Out[289]=
$$4 \left(w^2 + x^2 - y^2 - z^2\right) \left(2 + w^2 + x^2 + y^2 + z^2\right)$$

In[290]:=
```
realpoly = FullSimplify[Re[charpoly], {w^2 + x^2 == y^2 + z^2,
    Element[w, Reals], Element[x, Reals], Element[y, Reals], Element[z, Reals]}];
```

In[291]:=
```
realpoly = ReplaceAll[realpoly, {x → Sqrt[-w^2 + y^2 + z^2]}];
```

In[292]:=
```
realpoly =
 FullSimplify[realpoly, {Element[w, Reals], Element[y, Reals], Element[z, Reals]}]
```

Out[292]=
$$-4 + 16 \left(-w^4 + y^8 + z^4 \left(1 + z^2\right)^2 + w^2 \left(y^2 + z^2\right) + y^6 \left(2 + 4 z^2\right) + y^2 z^2 \left(3 + 6 z^2 + 4 z^4\right) + y^4 \left(1 + 6 \left(z^2 + z^4\right)\right)\right)$$

In[293]:=
```
altpoly = ReplaceAll[realpoly, {w → r * Cos[th], y → r * Cos[phi], z → r * Sin[phi]}];
```

In[294]:=
```
altpoly = FullSimplify[altpoly, {r > 0, Element[th, Reals], Element[phi, Reals]}];
```

In[295]:=
```
Collect[ExpandAll[altpoly], r]
```

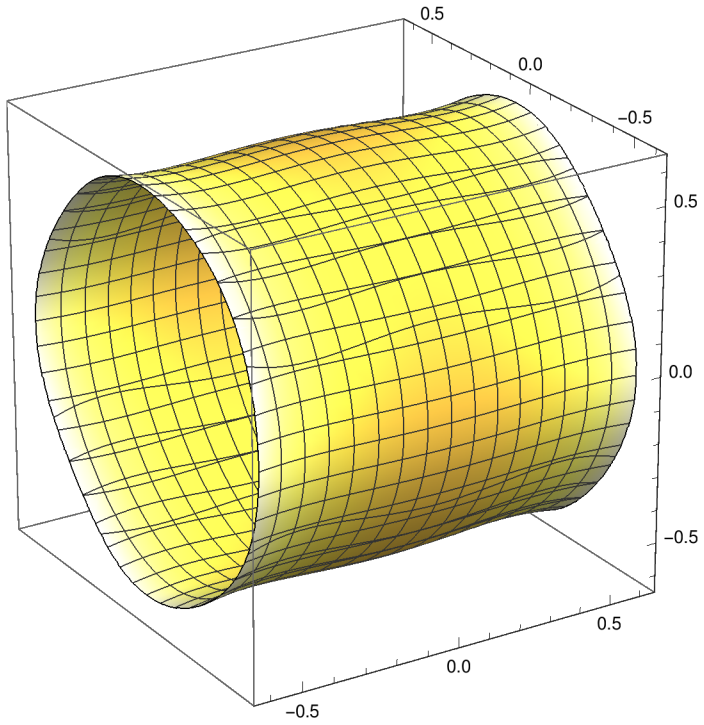
Out[295]=
$$-4 + 32 r^6 + 16 r^8 + r^4 \left(20 - 2 \text{Cos}[4 \text{ phi}] - 2 \text{Cos}[4 \text{ th}]\right)$$

In[296]:=
```
Collect[Expand[D[altpoly, r]], r]
```

Out[296]=
$$192 r^5 + 128 r^7 + r^3 \left(80 - 8 \text{Cos}[4 \text{ phi}] - 8 \text{Cos}[4 \text{ th}]\right)$$

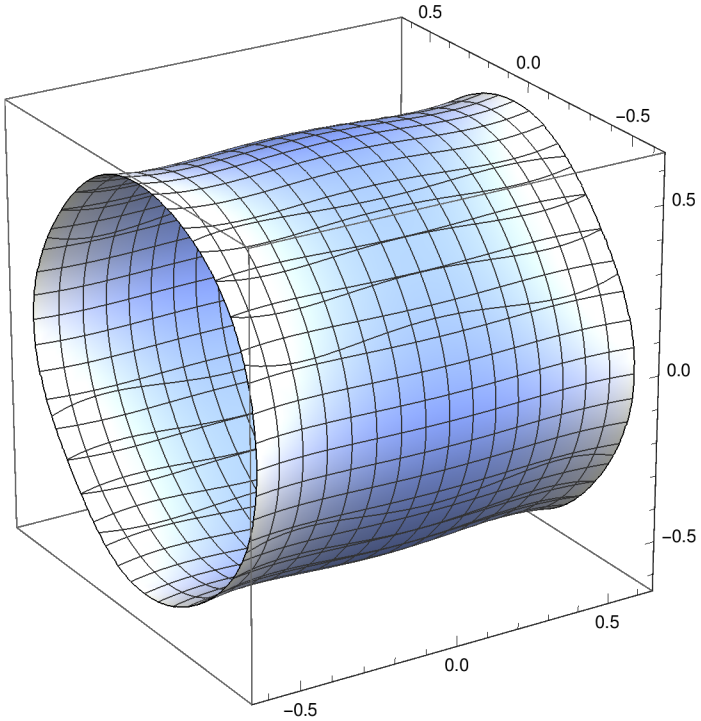
In[297]:=
```
upPlot = ContourPlot3D[realpoly == 0, {w, -5/8, 5/8}, {y, -5/8, 5/8},
  {z, -5/8, 5/8}, RegionFunction → Function[{w, y, z}, 0 < y^2 + z^2 - w^2],
  ColorFunction → Function[{w, y, z},
    ColorData["TemperatureMap"][0.5 + 0.5 Sqrt[y^2 + z^2 - w^2]]],
  ColorFunctionScaling → False, ViewPoint → {-5, -8, 4},
  BoxRatios → Automatic, PlotPoints → 25, MaxRecursion → 3]
```

Out[297]=



In[298]:=
```
downPlot = ContourPlot3D[realpoly == 0, {w, -5/8, 5/8}, {y, -5/8, 5/8},
  {z, -5/8, 5/8}, RegionFunction → Function[{w, y, z}, 0 < y^2 + z^2 - w^2],
  ColorFunction → Function[{w, y, z},
    ColorData["TemperatureMap"][0.5 - 0.5 * Sqrt[y^2 + z^2 - w^2]]],
  ColorFunctionScaling → False, ViewPoint → {-5, -8, 4},
  BoxRatios → Automatic, PlotPoints → 25, MaxRecursion → 3]
```

Out[298]=



In[299]:=
```
Export["torus_4matrices_n4_up.eps", upPlot, ImageSize → 3.2 * 72];
```

In[300]:=
```
Export["torus_4matrices_n4_down.eps", downPlot, ImageSize → 3.2 * 72];
```