In[1]:=
```
(*Scaling sigma_x and sigma_z of the Pauli Spin matrices with n=
 1/2 give us the lemniscate. This image is used to produce
  Lemniscate1.eps .*)
```

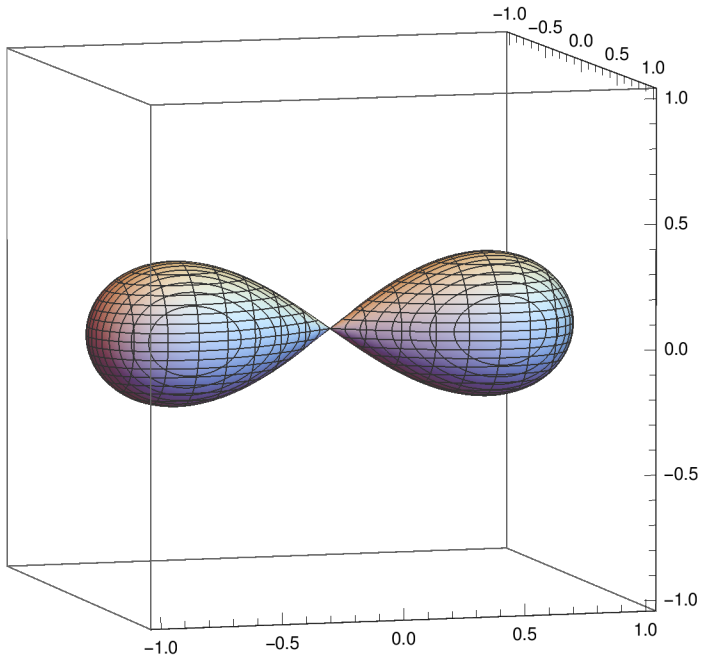
In[2]:=
```
n = 1 / 2;
sigma1 = {{0, 1}, {1, 0}};
sigma2 = {{0, -I}, {I, 0}};
sigma3 = {{1, 0}, {0, -1}};
loclzr = KroneckerProduct[sigma1, n * sigma1 - x * IdentityMatrix[2]] +
    KroneckerProduct[sigma2, sigma2 - y * IdentityMatrix[2]] +
    KroneckerProduct[sigma3, n * sigma3 - z * IdentityMatrix[2]] ;
MatrixForm[loclzr]
charpoly = FullSimplify[Det[loclzr]]
pl = ContourPlot3D[charpoly == 0, {x, -1, 1}, {y, -1, 1}, {z, -1, 1},
  Contours → {{1, LightBlue}}, PlotPoints → 100, ViewPoint → {72, -21, 9}]
```

Out[7]//MatrixForm=

$$\begin{pmatrix} \frac{1}{2} - z & 0 & -x + i\,y & -\frac{1}{2} \\ 0 & -\frac{1}{2} - z & \frac{3}{2} & -x + i\,y \\ -x - i\,y & \frac{3}{2} & -\frac{1}{2} + z & 0 \\ -\frac{1}{2} & -x - i\,y & 0 & \frac{1}{2} + z \end{pmatrix}$$

Out[8]=
$$x^4 - y^2 + 2\,z^2 + \left(y^2 + z^2\right)^2 + 2\,x^2\,\left(1 + y^2 + z^2\right)$$

Out[9]=



In[10]:=
```
Export["Lemniscate1.eps", pl, ImageSize → 3.2 * 72]
```

Out[10]= Lemniscate1.eps

In[11]:=
```
A = ReplaceAll[loclzr, {x → 0, y → 1/2, z → 0}]
```

Out[11]=
$$\left\{\left\{\frac{1}{2}, 0, \frac{i}{2}, -\frac{1}{2}\right\}, \left\{0, -\frac{1}{2}, \frac{3}{2}, \frac{i}{2}\right\}, \left\{-\frac{i}{2}, \frac{3}{2}, -\frac{1}{2}, 0\right\}, \left\{-\frac{1}{2}, -\frac{i}{2}, 0, \frac{1}{2}\right\}\right\}$$

In[12]:=
```
MatrixForm[A]
```

Out[12]//MatrixForm=
$$\begin{pmatrix} \frac{1}{2} & 0 & \frac{i}{2} & -\frac{1}{2} \\ 0 & -\frac{1}{2} & \frac{3}{2} & \frac{i}{2} \\ -\frac{i}{2} & \frac{3}{2} & -\frac{1}{2} & 0 \\ -\frac{1}{2} & -\frac{i}{2} & 0 & \frac{1}{2} \end{pmatrix}$$

In[13]:=
```
Eigenvalues[A]
```

Out[13]=
$$\left\{\frac{1}{2}\left(-2-\sqrt{5}\right), \frac{3}{2}, \frac{1}{2}, \frac{1}{2}\left(-2+\sqrt{5}\right)\right\}$$

In[14]:=
```
B = ReplaceAll[loclzr, {x → 0, y → -1/2, z → 0}]
```

Out[14]=
$$\left\{\left\{\frac{1}{2}, 0, -\frac{i}{2}, -\frac{1}{2}\right\}, \left\{0, -\frac{1}{2}, \frac{3}{2}, -\frac{i}{2}\right\}, \left\{\frac{i}{2}, \frac{3}{2}, -\frac{1}{2}, 0\right\}, \left\{-\frac{1}{2}, \frac{i}{2}, 0, \frac{1}{2}\right\}\right\}$$

In[15]:=
```
MatrixForm[B]
```

Out[15]//MatrixForm=
$$\begin{pmatrix} \frac{1}{2} & 0 & -\frac{i}{2} & -\frac{1}{2} \\ 0 & -\frac{1}{2} & \frac{3}{2} & -\frac{i}{2} \\ \frac{i}{2} & \frac{3}{2} & -\frac{1}{2} & 0 \\ -\frac{1}{2} & \frac{i}{2} & 0 & \frac{1}{2} \end{pmatrix}$$

In[16]:=
```
Factor[CharacteristicPolynomial[A, x]]
```

Out[16]=
$$\frac{1}{16}\left(-3+2x\right)\left(-1+2x\right)\left(-1+8x+4x^2\right)$$

In[17]:=
```
Eigenvalues[B]
```

Out[17]=
$$\left\{\frac{1}{2}\left(-2-\sqrt{5}\right), \frac{3}{2}, \frac{1}{2}, \frac{1}{2}\left(-2+\sqrt{5}\right)\right\}$$

In[18]:=
```
Factor[CharacteristicPolynomial[B, x]]
```

Out[18]=
$$\frac{1}{16}\left(-3+2x\right)\left(-1+2x\right)\left(-1+8x+4x^2\right)$$