

In[40]:= **n = 4;**

In[41]:= **q = 2;**

In[42]:= **r = 3/2;**

In[43]:= **s = 1;**

In[44]:= **t = 1;**

In[45]:= **sigmax = {{0, 1}, {1, 0}};**

In[46]:= **sigmay = {{0, -i}, {i, 0}};**

In[47]:= **sigmaz = {{1, 0}, {0, -1}};**

In[48]:= **I2 = IdentityMatrix[2];**

In[49]:= **AA = q * KroneckerProduct[sigmaz, sigmax];**

In[50]:= **MatrixForm[AA]**

Out[50]/MatrixForm=

$$\begin{pmatrix} 0 & 2 & 0 & 0 \\ 2 & 0 & 0 & 0 \\ 0 & 0 & 0 & -2 \\ 0 & 0 & -2 & 0 \end{pmatrix}$$

In[51]:= **BB = {{r, -i, 0, 0}, {i, 0, 0, 0}, {0, 0, 0, i}, {0, 0, -i, r}};**

In[52]:= **MatrixForm[BB]**

Out[52]/MatrixForm=

$$\begin{pmatrix} \frac{3}{2} & -i & 0 & 0 \\ 2 & 0 & 0 & 0 \\ i & 0 & 0 & 0 \\ 0 & 0 & 0 & i \\ 0 & 0 & -i & \frac{3}{2} \end{pmatrix}$$

In[53]:= **CC = s * KroneckerProduct[sigmaz, sigmaz];**

In[54]:= **MatrixForm[CC]**

Out[54]/MatrixForm=

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

In[55]:= **DD = t * KroneckerProduct[sigmax, I2];**

In[56]:= **MatrixForm[DD]**

Out[56]/MatrixForm=

$$\begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix}$$

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In[57]:= loclzrHalf = KroneckerProduct[i * sigmax, AA - w * IdentityMatrix[4]] +
  KroneckerProduct[i * sigmay, BB - x * IdentityMatrix[4]] +
  KroneckerProduct[i * sigmaz, CC - y * IdentityMatrix[4]] +
  KroneckerProduct[I2, DD - z * IdentityMatrix[4]];
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In[58]:= charpoly = FullSimplify[Det[loclzrHalf]]
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Out[58]= 
$$\frac{1}{16} \left( 16 w^8 - 96 x^7 + 16 x^8 + y \left( 3348 + y \left( 3294 + y \left( 1764 + y \left( 2121 + 8 y \left( 18 + 49 y + 2 y^3 \right) \right) \right) \right) \right) \right) +$$


$$2 y \left( 882 + y \left( 2121 + 4 y \left( 36 + 147 y + 8 y^3 \right) \right) \right) z^2 + 3 \left( 707 + 8 y \left( 6 + 49 y + 4 y^3 \right) \right) z^4 +$$


$$8 \left( 49 + 8 y^2 \right) z^6 + 16 z^8 + 8 x^6 \left( 67 + 8 y^2 + 8 z^2 \right) + 8 w^6 \left( 1 + 4 x \left( -3 + 2 x \right) + 8 y^2 + 8 z^2 \right) -$$


$$24 x^5 \left( 69 + 4 y \left( 2 + 3 y \right) + 12 z^2 \right) + 9 \left( -63 + 74 z^2 \right) -$$


$$48 x^3 \left( 105 + 58 y + 69 y^2 + 8 y^3 + 6 y^4 + \left( 69 + 4 y \left( 2 + 3 y \right) \right) z^2 + 6 z^4 \right) +$$


$$2 x^2 \left( 2133 + 2322 y + 3129 y^2 + 432 y^3 + 660 y^4 + 32 y^6 + 3 \left( 947 + 8 y \left( 18 + 55 y + 4 y^3 \right) \right) z^2 + \right.$$


$$\left. 12 \left( 55 + 8 y^2 \right) z^4 + 32 z^6 \right) + 24 x \left( -y \left( 276 + y \left( 246 + y \left( 116 + y \left( 69 + 4 y \left( 2 + y \right) \right) \right) \right) \right) \right) -$$


$$2 y \left( 58 + y \left( 69 + 8 y + 6 y^2 \right) \right) z^2 - \left( 69 + 4 y \left( 2 + 3 y \right) \right) z^4 - 4 z^6 - 3 \left( 27 + 70 z^2 \right) \right) +$$


$$3 x^4 \left( 1187 + 488 z^2 + 8 \left( 30 y + 4 y^4 + 4 z^4 + y^2 \left( 61 + 8 z^2 \right) \right) \right) +$$


$$3 w^4 \left( -93 + 48 y + 136 z^2 + 8 \left( -12 x^3 + 4 x^4 + 4 y^4 + 4 z^4 + \right. \right.$$


$$\left. \left. y^2 \left( 17 + 8 z^2 \right) + x^2 \left( 23 + 8 y^2 + 8 z^2 \right) - x \left( 21 + 8 y + 12 y^2 + 12 z^2 \right) \right) \right) +$$


$$2 w^2 \left( 477 - 144 x^5 + 32 x^6 + y \left( 18 + y \left( -231 + 4 y \left( 36 + 99 y + 8 y^3 \right) \right) \right) - 231 z^2 + \right.$$


$$24 y \left( 6 + 33 y + 4 y^3 \right) z^2 + 12 \left( 33 + 8 y^2 \right) z^4 + 32 z^6 +$$


$$12 x^4 \left( 45 + 8 y^2 + 8 z^2 \right) - 24 x^3 \left( 45 + 4 y \left( 2 + 3 y \right) + 12 z^2 \right) -$$


$$24 x \left( -15 + 10 y + 45 y^2 + 8 y^3 + 6 y^4 + \left( 45 + 4 y \left( 2 + 3 y \right) \right) z^2 + 6 z^4 \right) +$$


$$\left. \left. 3 x^2 \left( 163 + 312 z^2 + 8 \left( 18 y + 4 y^4 + 4 z^4 + y^2 \left( 39 + 8 z^2 \right) \right) \right) \right) \right)$$

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In[59]:= Factor[charpoly]
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Out[59]= 
$$\frac{1}{16}$$


$$\left( -567 + 954 w^2 - 279 w^4 + 8 w^6 + 16 w^8 - 1944 x + 720 w^2 x - 504 w^4 x - 96 w^6 x + 4266 x^2 + 978 w^2 x^2 + \right.$$


$$552 w^4 x^2 + 64 w^6 x^2 - 5040 x^3 - 2160 w^2 x^3 - 288 w^4 x^3 + 3561 x^4 + 1080 w^2 x^4 + 96 w^4 x^4 -$$


$$1656 x^5 - 288 w^2 x^5 + 536 x^6 + 64 w^2 x^6 - 96 x^7 + 16 x^8 + 3348 y + 36 w^2 y + 144 w^4 y - 6624 x y -$$


$$480 w^2 x y - 192 w^4 x y + 4644 x^2 y + 864 w^2 x^2 y - 2784 x^3 y - 384 w^2 x^3 y + 720 x^4 y -$$


$$192 x^5 y + 3294 y^2 - 462 w^2 y^2 + 408 w^4 y^2 + 64 w^6 y^2 - 5904 x y^2 - 2160 w^2 x y^2 - 288 w^4 x y^2 +$$


$$6258 x^2 y^2 + 1872 w^2 x^2 y^2 + 192 w^4 x^2 y^2 - 3312 x^3 y^2 - 576 w^2 x^3 y^2 + 1464 x^4 y^2 + 192 w^2 x^4 y^2 -$$


$$288 x^5 y^2 + 64 x^6 y^2 + 1764 y^3 + 288 w^2 y^3 - 2784 x y^3 - 384 w^2 x y^3 + 864 x^2 y^3 - 384 x^3 y^3 +$$


$$2121 y^4 + 792 w^2 y^4 + 96 w^4 y^4 - 1656 x y^4 - 288 w^2 x y^4 + 1320 x^2 y^4 + 192 w^2 x^2 y^4 - 288 x^3 y^4 +$$


$$96 x^4 y^4 + 144 y^5 - 192 x y^5 + 392 y^6 + 64 w^2 y^6 - 96 x y^6 + 64 x^2 y^6 + 16 y^8 + 666 z^2 - 462 w^2 z^2 +$$


$$408 w^4 z^2 + 64 w^6 z^2 - 5040 x z^2 - 2160 w^2 x z^2 - 288 w^4 x z^2 + 5682 x^2 z^2 + 1872 w^2 x^2 z^2 +$$


$$192 w^4 x^2 z^2 - 3312 x^3 z^2 - 576 w^2 x^3 z^2 + 1464 x^4 z^2 + 192 w^2 x^4 z^2 - 288 x^5 z^2 + 64 x^6 z^2 +$$


$$1764 y z^2 + 288 w^2 y z^2 - 2784 x y z^2 - 384 w^2 x y z^2 + 864 x^2 y z^2 - 384 x^3 y z^2 + 4242 y^2 z^2 +$$


$$1584 w^2 y^2 z^2 + 192 w^4 y^2 z^2 - 3312 x y^2 z^2 - 576 w^2 x y^2 z^2 + 2640 x^2 y^2 z^2 + 384 w^2 x^2 y^2 z^2 -$$


$$576 x^3 y^2 z^2 + 192 x^4 y^2 z^2 + 288 y^3 z^2 - 384 x y^3 z^2 + 1176 y^4 z^2 + 192 w^2 y^4 z^2 - 288 x y^4 z^2 +$$


$$192 x^2 y^4 z^2 + 64 y^6 z^2 + 2121 z^4 + 792 w^2 z^4 + 96 w^4 z^4 - 1656 x z^4 - 288 w^2 x z^4 + 1320 x^2 z^4 +$$


$$192 w^2 x^2 z^4 - 288 x^3 z^4 + 96 x^4 z^4 + 144 y z^4 - 192 x y z^4 + 1176 y^2 z^4 + 192 w^2 y^2 z^4 -$$


$$\left. \left. 288 x y^2 z^4 + 192 x^2 y^2 z^4 + 96 y^4 z^4 + 392 z^6 + 64 w^2 z^6 - 96 x z^6 + 64 x^2 z^6 + 64 y^2 z^6 + 16 z^8 \right) \right)$$

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```
In[60]:= localizerHalfAtZero = ReplaceAll[loclzrHalf, z → 0];
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```
In[61]:= gma =
```

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    KroneckerProduct[IdentityMatrix[2], KroneckerProduct[sigmaz, IdentityMatrix[2]]];
```

```
In[62]:= MatrixForm[gma];
```

```
In[63]:= madeHermitian = ExpandAll[i * localizerHalfAtZero.gma];
```

```
In[64]:= MatrixForm[madeHermitian]
```

```
Out[64]/MatrixForm=
```

$$\begin{pmatrix} -1+y & 0 & -i & 0 & \frac{3i}{2}+w-i x & -1 & 0 & 0 \\ 0 & 1+y & 0 & -i & -3 & w-i x & 0 & 0 \\ i & 0 & -1-y & 0 & 0 & 0 & -w+i x & -1 \\ 0 & i & 0 & 1-y & 0 & 0 & -3 & -\frac{3i}{2}-w+i x \\ -\frac{3i}{2}+w+i x & -3 & 0 & 0 & 1-y & 0 & -i & 0 \\ -1 & w+i x & 0 & 0 & 0 & -1-y & 0 & -i \\ 0 & 0 & -w-i x & -3 & i & 0 & 1+y & 0 \\ 0 & 0 & -1 & \frac{3i}{2}-w-i x & 0 & i & 0 & -1+y \end{pmatrix}$$

```
In[65]:= kTheory = FullSimplify[Det[madeHermitian]]
```

```
Out[65]= w8 + w6  $\left(\frac{1}{2} - 6x + 4x^2 + 4y^2\right) +$ 
```

$$\frac{1}{8} w^2 (x (-3 + 2x) (-120 + x (-3 + 2x) (81 + 4x (-3 + 2x)))) - 48 (-1 + x) x (-5 + 4x) y +$$

$$3 (-77 + 8x (-3 + 2x) (15 + x (-3 + 2x))) y^2 + 48 (3 - 4x) y^3 +$$

$$12 (33 + 4x (-3 + 2x)) y^4 + 32 y^6 + 9 (53 + 2y) +$$

$$\frac{3}{16} w^4 (-93 + 48y + 8 (x (-3 + 2x) (7 + x (-3 + 2x)) - 8xy + (17 + 4x (-3 + 2x)) y^2 + 4y^4)) +$$

$$\frac{1}{16} (-3 - 12x^3 + 4x^4 - 12x(1+y)^2 + x^2(17 + 8y^2) + y(18 + 17y + 4y^3))$$

$$(189 - 12x^3 + 4x^4 + x^2(81 + 8y^2) + y(18 + 81y + 4y^3) - 12x(9 + y(2 + y)))$$

```
In[66]:= (1/2) * Total[Sign[Eigenvalues[N[ReplaceAll[madeHermitian, {w → 0, x → 0, y → 0}]]]]]
```

```
Out[66]= -1
```

```
In[67]:= (1/2) *
```

```
    Total[Sign[Eigenvalues[N[ReplaceAll[madeHermitian, {w → 0.1, x → 0, y → 0}]]]]]
```

```
Out[67]= -1
```

```
In[68]:= (1/2) * Total[Sign[Eigenvalues[N[ReplaceAll[madeHermitian, {w → 0, x → 2, y → 0}]]]]]
```

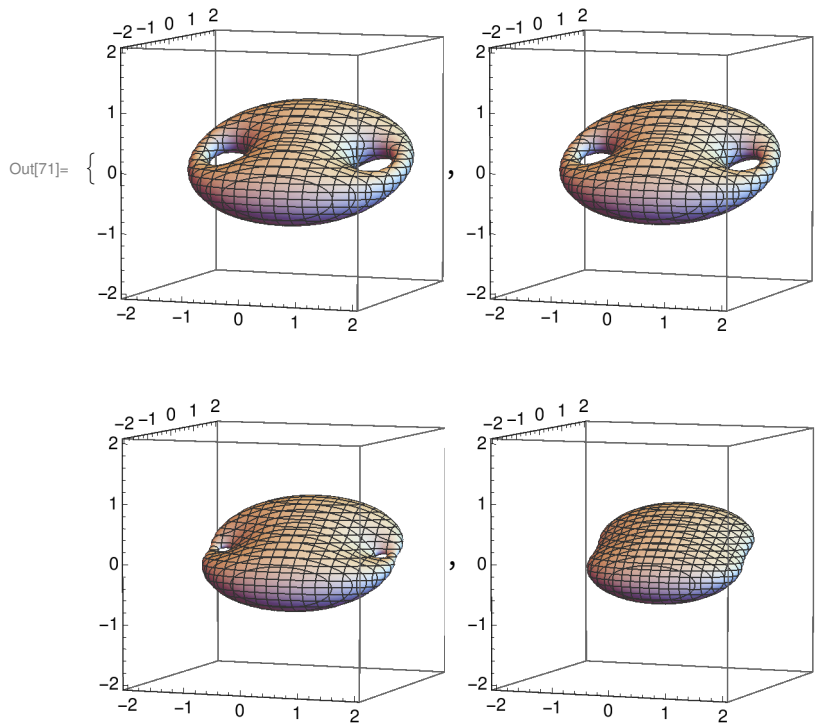
```
Out[68]= 0
```

```
In[69]:= (1/2) * Total[Sign[Eigenvalues[N[ReplaceAll[madeHermitian, {w → 2, x → 0, y → 2}]]]]]
```

```
Out[69]= 0
```

```
In[70]:= step = 1/5;
```

```
In[71]:= plots = ParallelTable[ContourPlot3D[charpoly == 0,
  {w, -2, 2}, {x, -2, 2}, {y, -2, 2}, Contours -> {{1, LightBlue}},
  PlotPoints -> 100, ViewPoint -> {7, -18, 2}], {z, 0, 3 * step, step}]
```



```
In[72]:=
```

```
In[73]:= Export["AIII_class_sphere_deform_r_3_halves_z_1.eps",
  plots[[1]], ImageSize -> 3.2 * 72];
```

```
In[74]:= Export["AIII_class_sphere_deform_r_3_halves_z_2.eps",
  plots[[2]], ImageSize -> 3.2 * 72];
```

```
In[75]:= Export["AIII_class_sphere_deform_r_3_halves_z_3.eps",
  plots[[3]], ImageSize -> 3.2 * 72];
```

```
In[76]:= Export["AIII_class_sphere_deform_r_3_halves_z_4.eps",
  plots[[4]], ImageSize -> 3.2 * 72];
```