Math 563, Fall 2016
Assignment 8, due Friday, December 9

1. (Convolution workout) Begin by working through parts (a), (b), (c) in Stein & Shakarchi, Chapter 2, Exercise #21 on your own (i.e. you do not need to hand them in). Then hand in the following problems:

   (a) Show that $\text{supp}(f * g) \subset \text{supp}(f) + \text{supp}(g)$.

   (b) Show that $f * g$ is uniformly continuous when f is integrable and g is continuous and bounded.

   (c) Show that if f and g are both integrable on $\mathbb{R}^d$, then $(f * g)(x) \to 0$ as $|x| \to \infty$. (Hint: by part (a), this would be clear if $\text{supp}(f)$ and $\text{supp}(g)$ were bounded. While you can't assume this, integrable functions "almost" have bounded support by Proposition 1.12 in Ch. 2.)

   (d) Prove Young's inequality: Suppose $1 \leq p \leq \infty$. If $f \in L^p(\mathbb{R}^d)$, $g \in L^1(\mathbb{R}^d)$, then $f * g \in L^p(\mathbb{R}^d)$ with

   $$\|f * g\|_{L^p(\mathbb{R}^d)} \leq \|f\|_{L^p(\mathbb{R}^d)} \|g\|_{L^1(\mathbb{R}^d)}.$$

   Hint: At least in principle, the left hand side of the inequality is

   $$\sup \left\{ \left| \int (f * g)(x) h(x)\, dx \right| : \|h\|_{L^{p'}(\mathbb{R}^d)} = 1 \right\}.$$

   You will have to treat the case $p = \infty$ separately.

2. Stein-Shakarchi, Exercise #1, Chapter 3.

3. Stein-Shakarchi, Exercise #7, Chapter 3.

4. Stein-Shakarchi, Exercise #11, Chapter 3.

Reading: Stein-Shakarchi, Ch. 3, sections 1-3.

On your own: Stein-Shakarchi, Exercises #4 and #14, Chapter 3.